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WHITE PAPER

AN EXPLANATION OF THE BETA AND STEINHART-HART EQUATIONS FOR REPRESENTING THE RESISTANCE VS. TEMPERATURE RELATIONSHIP IN NTC THERMISTOR MATERIALS

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ABSTRACT

This paper discusses the comparison of two equations typically used for interpolating the resistance vs. temperature characteristic of NTC (Negative Temperature Coefficient) thermistors: The Beta Equation and the Steinhart-Hart Equation. The information presented illustrates the deficiencies of the Beta Equation in terms of calculating the R/T characteristic of NTC thermistors for tight tolerance devices that are used for applications with temperature spans greater than 20°C. In addition, the advantages of using the Steinhart-Hart Equation for obtaining greatly reduced uncertainties of resistance vs. temperature data over wider temperature spans are shown.

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The early scientific literature regarding the resistance vs. temperature relationship for Negative Temperature Coefficient (NTC) thermistors proposed the assumption that these materials followed the model in conductivity physics known as intrinsic conduction, which is represented by the following linear mathematical expression [1]:

$$\ln R_T = A + \beta/T \qquad (1)$$

where T = absolute temperature (Kelvin) and β (Beta) is assumed to be a material constant (Kelvin).

By introducing an equation that describes a thermistor with a resistance value at a standard reference temperature,

$$\ln R_{T0} = A + \beta/T_0 \qquad (2)$$

and by solving the simultaneous set of equations (1) and (2) for resistance, we obtain the following expressions:

$$\ln R_T - \ln R_{T0} = \ln(R_T/R_{T0}) = \beta(1/T-1/T_0) \qquad (3)$$

or $$R_T = R_{T0} \exp\{ \beta(1/T-1/T_0)\} \qquad (4)$$

Thus: $$\beta = \ln(R_T/R_{T0})/(1/T-1/T_0) \qquad (5)$$

and $$T = [(1/\beta) \ln(R_T/R_{T0}) + 1/T_0]^{-1} \qquad (6)$$

Therefore, if two points or a single point and the slope of the resistance/temperature characteristic are known, the equation can be solved for any unknown quantity [2,3].

In the past, these expressions were adequate for applications requiring narrow temperature spans of 20°C or less. However, with the improvements in technology for temperature calibration, the early expression, known as the Beta Equation, was found to be inadequate for applications requiring more precise temperature measurement over broader temperature spans of 50°C or more. After further study, it was determined that Beta is dependent on temperature and is not a true constant. Therefore, the need for an improved resistance/temperature curve fit model based on the conductivity physics property known as extrinsic conduction became apparent.

Researchers have attempted to provide improved empirical equations that take into account the temperature dependence of Beta, but solutions for many of these equations were more difficult and not widely accepted for commercial thermistor applications. In their search for an empirical expression that provided relatively accurate interpolation of the resistance/temperature characteristic in the oceanographic temperature range of -2°C to 35°C, Steinhart and Hart investigated the curve fitting capability of the following Equation (7) [1,3]:

$$1/T = A + B(\ln R) + C(\ln R)^2 + D(\ln R)^3 \qquad (7)$$

and later discovered that by eliminating the squared term, the following equation

$$1/T = A + B(\ln R) + C(\ln R)^3 \qquad (8)$$

improved the curve-fitting results for the ocean water temperatures they were studying. This latter, more popular form of the equation became known as the Steinhart-Hart Equation.

Equation (7) has been studied by various other researchers, including scientists at NIST, who have determined that this form is advantageous for curve fitting thermistors over 100°C spans between -80°C and 260°C, 150°C spans between -60°C and 260°C, and 150°C to 200°C spans between 0°C and 260°C. Within these parameters, the interpolation errors associated with using Equation (7) have been shown to be less than or equal to the total measurement uncertainties of the four resistance/temperature data points needed to solve for the four temperature coefficients used in the equation. These results were confirmed by comparing the thermistor data to Standard Platinum Resistance Thermometers (SPRTs) and fixed-point cell temperature standards based on the ITS-90 temperature scale. By using test equipment with very low uncertainty capabilities, the resistance/temperature characteristics of the thermistors were found to be in agreement on the order of milli Kelvins (mK) with the other temperature standards [4].

Equation (8) has been a more widely used form of the equation since the solutions for the three coefficients are more easily attained with three resistance/temperature data points. For this equation, typically the interpolation errors are less than 0.004°C for 50°C spans and 0.010°C for 100°C spans at temperatures greater than 0°C. Greater interpolation errors (on the order of 0.010 °C to 0.050°C) can occur for either wider spans (100°C to 150°C) and/or the extremely low temperature (-80°C to 0°C) or extremely high temperature (100°C to 260°C) regions of the temperature spectrum [2]. To overcome these limitations, many users will typically limit the temperature spans to 50°C and calculate the resistance/temperature data points in 50°C sections over the entire temperature range for the application.

To illustrate the difference in interpolation errors between the Beta and Steinhart-Hart equations, known data points from QTI Sensing Solutions Curve E R/T Table were used to calculate resistance vs. temperature in 1°C increments over the range of 0°C to 50°C. The data were analyzed by comparing the two sets of calculations and showing the difference in resistance % and °C between the data at each temperature point. Table 1 below illustrates the results of this comparison. The column with NTC values was provided to show the relationship between the percent resistance deviation and temperature deviation of the Beta Equation calculations. The temperature deviation data were calculated by dividing the percent resistance by the NTC.

Table 1. Comparison of Resistance Calculations Derived from Beta and Steinhart-Hart Equations for QTI Sensing Solutions R/T Curve E.

Temp (°C)	Temp (K)	R/T Data Steinhart-Hart Equation (Ω)	R/T Calc from Beta Equation β = 3811K (Ω)	Deviation of Beta R/T Data from Steinhart-Hart R/T Data		NTC (%/°C)
				(%)	(°C)	
0	273.15	94980.00	94980.00	0.00	0.000	-4.94
1	274.15	90412.89	90267.22	-0.16	0.033	-4.91
2	275.15	86089.61	85820.02	-0.31	0.064	-4.89
3	276.15	81995.95	81621.78	-0.46	0.094	-4.86
4	277.15	78118.57	77657.01	-0.59	0.122	-4.83
5	278.15	74444.99	73911.30	-0.72	0.149	-4.80
6	279.15	70963.51	70371.17	-0.83	0.175	-4.78
7	280.15	67663.12	67024.09	-0.94	0.199	-4.75
8	281.15	64533.54	63858.34	-1.05	0.222	-4.72
9	282.15	61565.09	60862.98	-1.14	0.243	-4.70
10	283.15	58748.70	58027.82	-1.23	0.263	-4.67
11	284.15	56075.83	55343.30	-1.31	0.281	-4.64
12	285.15	53538.47	52800.52	-1.38	0.299	-4.62
13	286.15	51129.12	50391.13	-1.44	0.314	-4.59
14	287.15	48840.68	48107.33	-1.50	0.329	-4.57
15	288.15	46666.51	45941.82	-1.55	0.342	-4.54
16	289.15	44600.36	43887.77	-1.60	0.354	-4.52
17	290.15	42636.34	41938.78	-1.64	0.364	-4.49
18	291.15	40768.92	40088.85	-1.67	0.373	-4.47
19	292.15	38992.89	38332.35	-1.69	0.381	-4.44
20	293.15	37303.33	36664.02	-1.71	0.388	-4.42
21	294.15	35695.63	35078.91	-1.73	0.393	-4.39
22	295.15	34165.44	33572.39	-1.74	0.397	-4.37
23	296.15	32708.64	32140.09	-1.74	0.400	-4.35
24	297.15	31321.38	30777.93	-1.74	0.401	-4.32
25	298.15	30000.00	29482.06	-1.73	0.402	-4.30
26	299.15	28741.07	28248.88	-1.71	0.401	-4.28
27	300.15	27541.35	27074.98	-1.69	0.398	-4.25
28	301.15	26397.77	25957.19	-1.67	0.395	-4.23
29	302.15	25307.44	24892.48	-1.64	0.390	-4.21

Temp (°C)	Temp (K)	R/T Data Steinhart-Hart Equation (Ω)	R/T Calc from Beta Equation β = 3811K (Ω)	Deviation of Beta R/T Data from Steinhart-Hart R/T Data (%)	Deviation of Beta R/T Data from Steinhart-Hart R/T Data (°C)	NTC (%/°C)
30	303.15	24267.64	23878.05	-1.61	0.384	-4.18
31	304.15	23275.78	22911.23	-1.57	0.376	-4.16
32	305.15	22329.44	21989.50	-1.52	0.368	-4.14
33	306.15	21426.32	21110.52	-1.47	0.358	-4.12
34	307.15	20564.24	20272.06	-1.42	0.347	-4.10
35	308.15	19741.14	19472.03	-1.36	0.335	-4.07
36	309.15	18955.09	18708.44	-1.30	0.321	-4.05
37	310.15	18204.25	17979.43	-1.23	0.306	-4.03
38	311.15	17486.87	17283.24	-1.16	0.290	-4.01
39	312.15	16801.32	16618.21	-1.09	0.273	-3.99
40	313.15	16146.04	15982.78	-1.01	0.255	-3.97
41	314.15	15519.55	15375.46	-0.93	0.235	-3.95
42	315.15	14920.45	14794.86	-0.84	0.214	-3.93
43	316.15	14347.43	14239.64	-0.75	0.192	-3.91
44	317.15	13799.23	13708.57	-0.66	0.169	-3.89
45	318.15	13274.66	13200.46	-0.56	0.145	-3.87
46	319.15	12772.60	12714.19	-0.46	0.119	-3.85
47	320.15	12291.98	12248.71	-0.35	0.092	-3.83
48	321.15	11831.79	11803.01	-0.24	0.064	-3.81
49	322.15	11391.07	11376.14	-0.13	0.035	-3.79
50	323.15	10968.90	10967.21	-0.02	0.004	-3.77

Because the Beta Equation does not provide an adequate fit for a NTC thermistor resistance vs. temperature characteristic, the deviation from the nominal R/T curve can be relatively large - on the order of 0.4°C at the mid-point of the 0°C to 50°C range. The work of Steinhart-Hart was confirmed independently by B.W. Mangum at the National Institute of Standards and Technology (NIST), formerly known as the National Bureau of Standards (NBS), and R. Koehler at the Woods Hole Oceanographic Institute [4,5]. Therefore, in this illustration, we have used the Steinhart-Hart Equation as the interpolation standard as a means to calculate the resistance values in Table 1 as the reference points for the R/T characteristic.

To further illustrate the temperature dependence of β, Figures 1 and 2 below show the percentage error and comparable temperature error for resistance/temperature data over the temperature span of -50°C to 150°C using a typical R0/R50 β of 3811 K for QTI Sensing Solutions R/T Curve E.

Figure 1.

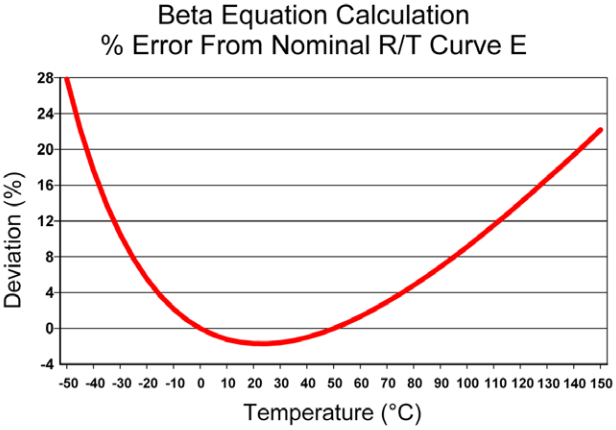
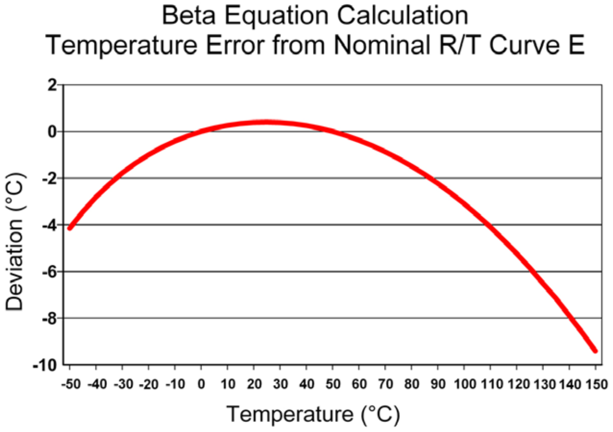


Figure 2.



Even after dividing the Beta Equation calculations of the R/T characteristic into 50°C spans, the errors from using this method are still too large. Figures 3 and 4 below illustrate this deficiency.

Figure 3.

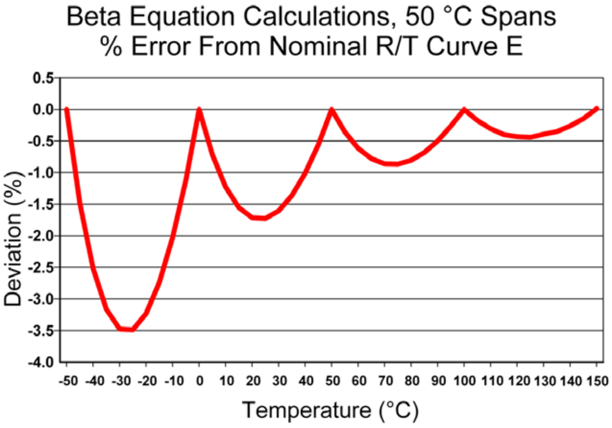
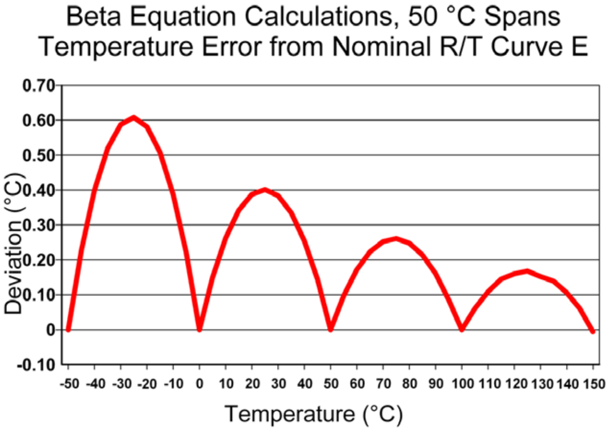


Figure 4.



Based on the illustrations above, the temperature deviations resulting from the Beta Equation calculations are too large for most modern temperature applications. For this reason, currently many NTC thermistor manufacturers' R/T tables are based on Steinhart-Hart Equations (7) or (8). For example, instead of using Beta tolerance as a means of specifying limits of the R/T curve-fit of NTC thermistors, QTI Sensing Solutions controls its thermistor manufacturing processes by establishing limits and specifications based on the applicable resistance ratios calculated from the R/T Tables derived from the Steinhart-Hart Equation. Unfortunately, some thermistor manufacturers still use Beta to define the tolerances on their specifications, so it can be confusing for end users to make valid comparisons between the various product offerings in the marketplace. Although trying to compare Beta tolerances to ratio tolerances can be like the proverbial expression of comparing "apples to oranges", QTI Sensing Solutions offers the following explanation to assist end users in designing the optimal thermistor solution for their application.

One of the primary concerns OEM customers and end-users typically have is determining how a thermistor manufacturer controls its production processes and what measurement methods and standards are used to ensure the end product meets the specified requirements. Some of the common parameters measured and controlled include electrical properties such as the R/T curve slope characteristic, resistance tolerance, temperature tolerance, time constant, dissipation constant and mechanical parameters such as coating material and size, and lead wire material and dimensions. For the purpose of this discussion, below we will concentrate on illustrating the means to evaluate the method of measuring and controlling the R/T curve slope characteristic, which is one of the most critical parameters listed. Unfortunately, many customer and manufacturing specifications still refer to Beta and the Beta tolerance to specify the R/T characteristic. As shown in Table 1, using Beta is not sufficient for properly representing the R/T characteristic for most tight-tolerance temperature applications and doing is like using a ruler with hash marks that vary in spacing along the length of the ruler.

As an example for defining a NTC thermistor specification, we will refer to a commonly used interchangeable NTC thermistor specification of ±0.2°C from 0°C to 70°C. The word interchangeable simply means the thermistor is manufactured to the specified tolerance over the temperature range and can be installed or replaced in a temperature measurement/control system without further calibration or circuit adjustment. Using interchangeable NTC thermistors helps the systems manufacturer or thermistor user reduce assembly labor costs as well as field replacement costs.

A properly manufactured interchangeable thermistor will be within the allowed temperature tolerance at any point within the specified temperature range. Reviewing the data from Table 1 illustrates a significant problem when using the Beta Equation to calculate the R/T values (for $\beta = 3811$ K), since the nominal resistance values at the mid-point of the 0°C to 50°C temperature range are significantly lower, (on the order of -1.74 %) than the values derived from the Steinhart-Hart Equation. These values correspond to a temperature deviation of 0.4°C, which is twice the allowable tolerance of ±0.2°C. Therefore, in this example, even if a manufacturer or end user specifies a Beta with a relatively tight tolerance (e.g. 3811 K ±0.33 %), which is equivalent to the ±0.2°C interchangeable tolerance at the end points, then the resistance values at the mid-point of the range will be significantly lower than the nominal R/T characteristic and well outside the allowable range.

By utilizing modern ceramic technology and maintaining strict process controls, QTI Sensing Solutions produces one of the most stable and reliable NTC chip thermistors available today. Since every step of the QTI Sensing Solutions NTC chip thermistor manufacturing process is carefully monitored and controlled, QTI is able to achieve excellent lot uniformity and batch-to-batch repeatability. By testing the R/T characteristics of the thermistor materials against the more accurate resistance ratio method based on the values derived from the Steinhart-Hart Equation (as opposed to Beta tolerances), QTI Sensing Solutions is able to offer tight tolerance interchangeable NTC thermistors for the most demanding applications.

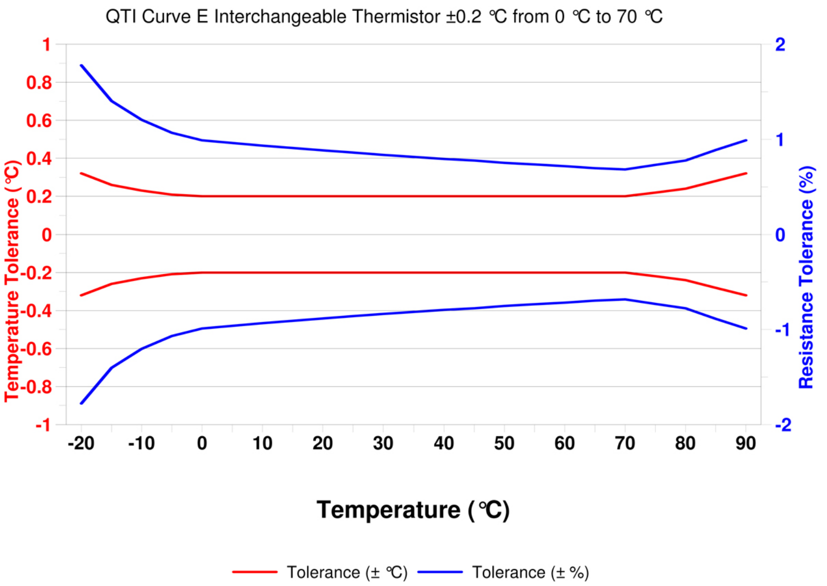
Table 2 below is an example of an interchangeable tolerance specification for a QTI Sensing Solutions thermistor with a R25 of 30,000 Ω, R/T Curve E. The data show the relationships between temperature tolerance, percent resistance tolerance, and the NTC used to calculate the resistance tolerance at each temperature. For a thermistor with ±0.2°C interchangeability from 0°C to 70°C, the resistance tolerance becomes tighter as the temperature increases. This phenomenon is caused by the NTC that decreases with increasing temperature. A common misconception among manufacturers and end users is to assume the resistance tolerance specified at 25°C is constant over the temperature range, but the data indicate that basing a specification on that assumption would potentially cause a thermistor to be out of tolerance at a higher temperature.

Table 2.
Example of an Interchangeable NTC Thermistor Specification
R25 = 30,000 ohms at 25°C, R/T Curve E, ±0.2°C from 0°C to 70°C

Temp (°C)	Resistance (Ω)	Temperature Tolerance (± °C)	NTC (%/°C)	Resistance Tolerance (± %)
0	94,980.0	0.2	-4.94	0.99
25	30,000.0	0.2	-4.30	0.86
70	5,357.4	0.2	-3.41	0.68

The graph in Figure 5 below further illustrates the relationships between these parameters. The left Y-axis and red line show the units in terms of temperature tolerance and the right Y-axis and blue line show the units in terms of percent resistance tolerance.

Figure 5.



Given the information presented above, this author suggests that the thermistor industry needs to modernize its approach and adopt a "new" standard for specifying the R/T characteristic of NTC thermistors. For today's demanding applications requiring tight tolerances over wider temperature spans, we can see that the Beta Equation simply is not adequate for the accurate representation of the NTC thermistor characteristic. For this reason, interchangeable NTC thermistors produced by QTI Sensing Solutions are specified, manufactured, and tested in such a manner to insure they will be within the required tolerance over the applicable temperature range. As mentioned previously, QTI's ability to produce consistent, reliable thermistors is a result of QTI's well-established, state-of-the-art, manufacturing process control and quality management systems.

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ABOUT QTI SENSING SOLUTIONS

QTI Sensing Solutions was founded in 1977 to meet the increasing demand for high quality electronic components for the aerospace industry. Since then, QTI has exceeded the requirements of some of the most stringent high cost of failure applications, changing the landscape of the supply chain for the entire industry.

Today, QTI continues to maintain its leadership position for mission-critical applications as well as for medical and industrial applications by supplying the world's top companies with innovative products and services. In fact, QTI developed the highest standard for surface mount thermistors with the introduction of qualified surface mount parts to MIL-PRF-32192; supplying design engineers with fully qualified Defense Logistics Agency options for two PTC and three NTC surface mount package styles. Additionally, QTI has partnered with the NASA Goddard Space Flight Center for surface mount thermistors qualified to S311-P827, an industry first!

In addition to QTI's accomplishments, our ISO:09001:2000 and AS9100 certified manufacturing and testing facilities in Idaho enhances our ability to meet the needs of today's challenging temperature measurement and control applications.

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